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Laminar forced convection inside ducts with periodic variation of inlet temperature

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Abstract—Laminar forced convection with periodic variations of inlet temperature is studied in both parallel-plate channels and circular ducts. The generalized integral transform technique is employed to reduce the original problem to a system of linear first-order differential equations, which is then solved utilizing the related complex matrix eigenvalue problem. Amplitudes and phase lags with respect to the inlet condition are determined for fluid bulk temperature and wall heat flux, and the results are presented in graphical form as a function of the dimensionless axial distance along the channels for different values of the dimensionless frequency of inlet oscillations.

INTRODUCTION

THE PERIODIC Graetz problem is of great interest in engineering applications related to the thermal response of heat exchanger equipment subjected to periodic disturbances on inlet temperature. The inherent difficulties associated with the analytical treatment of such problems have been pointed out in recent papers [1, 2].

It appears that Sparrow and Farias [3] were the first investigators to analytically study a problem of this type, for slug flow between parallel-plates, including wall conjugation effects. The resulting complex Sturm–Liouville system was solved by a trial and error procedure, and numerical difficulties were encountered in the evaluation of the complete spectrum of eigenquantities. Later, Kakaç and Yener [1, 4, 5] considered laminar and turbulent flow between parallel-plates subjected to periodic variations of inlet temperature, but without the conjugation with the walls. The related complex Sturm–Liouville system was more involved than that in Ref. [3], and an experimental investigation was employed to estimate the first set of eigenquantities.

The difficulties associated with the analytical solution of the periodic Graetz problem are strongly related to the accurate solution of the corresponding complex Sturm-Liouville system. In this work, we utilized a variation of the generalized integral transform technique discussed by Özişik and Murray [6] in order to alleviate the need for the complex eigenvalue problem. That is, we used the standard Sturm–Liouville system basic to the solution of the classical Graetz problem and transformed the original problem into a set of first-order linear ordinary differential equations. Such a system could be solved accurately by considering the related complex matrix eigensystem analysis.

ANALYSIS

We consider thermally developing, hydrodynamically developed laminar flow inside ducts, such as parallel-plate channels and circular tubes. The inlet temperature is assumed to vary periodically in time, and we seek the thermal response of the system to these periodic disturbances, after the initial transients have died out. Axial conduction, viscous dissipation, free convection, and wall conjugation effects are not taken into consideration, and physical properties assumed to be constant. Then the energy equation is given by:

$$\frac{\partial T(r,z,t)}{\partial t}+u(r)\frac{\partial T(r,z,t)}{\partial z}=\frac{\alpha}{r^n}\frac{\partial}{\partial r}\left(r^n\frac{\partial T(r,z,t)}{\partial r}\right),\quad \text{in } 0<r<b, z>0, t>0, \tag{1a}$$

where

$$n=\begin{cases}0, & \text{for parallel-plate channel}\\ 1, & \text{for circular duct,}\end{cases}$$

with inlet and boundary conditions are given, respectively, by:

$$T(r,0,t)=T_0+\Delta T_0 e^{i\omega t},\quad 0\leqslant r\leqslant b,\ t>0 \tag{1b}$$

$$\left.\frac{\partial T(r,z,t)}{\partial r}\right|_{r=0}=0,\quad z>0,\ t>0 \tag{1c}$$

$$T(b,z,t)=T_0,\qquad z>0,\ t>0. \tag{1d}$$

The following dimensionless groups are introduced

$R=\dfrac{r}{b}$, dimensionless normal coordinate

$Z=\dfrac{\alpha z}{\bar{u}b^2}$, dimensionless axial coordinate

NOMENCLATURE

- $\mathbf{A}$ matrix of coefficients in system (9)
- a_{kj} elements of coefficients matrix $\mathbf{A}$, equation (9e)
- a^*_{kj} defined by equation (8c)
- b radius of circular duct or half the spacing between parallel-plates
- $\mathbf{c}$ vector of constants, equation (13a)
- $\mathbf{f}$ vector defined by equation (9d)
- i $\sqrt{-1}$
- N order of matrix $\mathbf{A}$
- N_k normalization integral defined by equation (7)
- r radial (or normal) coordinate
- $T(r,z,t)$ temperature distribution
- T_0 mean temperature of inlet oscillations
- ΔT_0 amplitude of inlet oscillations
- t time variable
- $\bar{u}$ average flow velocity
- $\mathbf{v}$ eigenvectors of matrix $\mathbf{A}$
- $\mathbf{x}(Z)$ vector defined by equation (9c)
- z axial coordinate.

Greek symbols

- α thermal diffusivity of fluid
- $\tilde{\theta}(R,Z)$ transformed dimensionless temperature distribution
- $\tilde{\theta}_{av}(Z)$ transformed fluid bulk temperature
- λ eigenvalues of matrix $\mathbf{A}$
- μ_k eigenvalues of eigenvalue problem (5)
- $\psi(\mu,R)$ eigenfunctions of eigenvalue problem (5)
- $\Psi(Z)$ fundamental matrix of system (8)
- ω frequency of inlet oscillations.

Subscripts

- j column index in system (9)
- k row index in system (9) and order of eigenvalue
- l lowest-order solution.

Superscripts

- $\sim$ time transformation defined by equation (3)
- $-$ integral transform with respect to the R variable.

$\tau = \dfrac{\alpha t}{b^2}$, dimensionless time

$U(R) = \dfrac{u(r)}{\bar{u}}$, dimensionless velocity distribution

$\theta(R,Z,\tau) = \dfrac{T(r,z,t)-T_0}{\Delta T_0}$,

dimensionless temperature distribution

$\Omega = \dfrac{\omega b^2}{\alpha}$,

dimensionless frequency of inlet oscillations.

Equations (1) are then expressed in dimensionless form as:

$$\frac{\partial\theta(R,Z,\tau)}{\partial\tau} + U(R)\frac{\partial\theta(R,Z,\tau)}{\partial Z} = \frac{1}{R^n}\frac{\partial}{\partial R}\times\left[R^n\frac{\partial\theta(R,Z,\tau)}{\partial R}\right], \quad \text{in } 0<R<1,\ Z>0,\ \tau>0 \tag{2a}$$

$$\theta(R,0,\tau) = e^{i\Omega\tau}, \quad 0\leqslant R\leqslant 1,\ \tau>0 \tag{2b}$$

$$\left.\frac{\partial\theta(R,Z,\tau)}{\partial R}\right|_{R=0} = 0, \quad Z>0,\ \tau>0 \tag{2c}$$

$$\theta(1,Z,\tau) = 0, \quad Z>0,\ \tau>0. \tag{2d}$$

Since only the periodic response is of interest, we seek a solution in the form:

$$\theta(R,Z,\tau) = \tilde{\theta}(R,Z)\,e^{i\Omega\tau}, \tag{3}$$

which results in the following problem for $\tilde{\theta}(R,Z)$:

$$U(R)\frac{\partial\tilde{\theta}(R,Z)}{\partial Z} = \frac{1}{R^n}\frac{\partial}{\partial R}\left[R^n\frac{\partial\tilde{\theta}(R,Z)}{\partial R}\right] - i\Omega\tilde{\theta}(R,Z) \tag{4a}$$

$$\tilde{\theta}(R,0) = 1 \tag{4b}$$

$$\left.\frac{\partial\tilde{\theta}(R,Z)}{\partial R}\right|_{R=0} = 0 \tag{4c}$$

$$\tilde{\theta}(1,Z) = 0. \tag{4d}$$

A formal solution to this problem is a straightforward matter through the use of the classical integral transform technique [7]. However, the complete numerical solution would require an accurate evaluation of eigenvalues and eigenfunctions of the corresponding complex non-classical Sturm–Liouville system. To alleviate the difficulties in the solution of a complex eigenvalue problem, we choose to consider an auxiliary problem that is a special case of classical Sturm–Liouville systems:

$$\frac{d}{dR}\left[R^n\frac{d\psi(\mu_k,R)}{dR}\right] + \mu_k^2 R^n U(R)\psi(\mu_k,R) = 0, \quad \text{in } 0<R<1 \tag{5a}$$

$$\frac{d\psi(\mu_k, R)}{dR} = 0, \quad R = 0 \tag{5b}$$

$$\psi(\mu_k, R) = 0, \quad R = 1. \tag{5c}$$

By utilizing the eigenfunctions of this system, we define the following integral transform pair:

$$\text{inversion:} \quad \tilde{\theta}(R, Z) = \sum_{k=1}^{\infty} \frac{1}{N_k^{1/2}} \psi(\mu_k, R) \bar{\tilde{\theta}}_k(Z) \tag{6a}$$

$$\text{transform:} \quad \bar{\tilde{\theta}}_k(Z) = \int_0^1 R^n U(R) \frac{\psi(\mu_k, R)}{N_k^{1/2}} \tilde{\theta}(R, Z)\, dR, \tag{6b}$$

where the normalization integral is given by

$$N_k = \int_0^1 R^n U(R) [\psi(\mu_k, R)]^2\, dR. \tag{7}$$

We now operate on equation (4a) with the operator

$$\int_0^1 R^n \frac{\psi(\mu_k, R)}{N_k^{1/2}}\, dR,$$

and make use of boundary conditions to obtain:

$$\frac{d\bar{\tilde{\theta}}_k(Z)}{dZ} + \mu_k^2 \bar{\tilde{\theta}}_k(Z) + i\Omega \sum_{j=1}^{\infty} a_{ij}^* \bar{\tilde{\theta}}_j(Z) = 0, \tag{8a}$$

with the transformed inlet condition

$$\bar{\tilde{\theta}}_k(0) = \bar{f}_k = \int_0^1 R^n U(R) \frac{\psi(\mu_k, R)}{N_k^{1/2}}\, dR, \tag{8b}$$

where

$$a_{kj}^* = a_{jk}^* = \frac{1}{(N_k N_j)^{1/2}} \int_0^1 R^n \psi(\mu_k, R) \psi(\mu_j, R)\, dR. \tag{8c}$$

System (8) forms a set of infinite, coupled, first-order linear differential equations, which can be replaced by a finite number of coupled equations if a sufficiently large number of terms are considered in the summation appearing in equation (8a). Therefore, taking a sufficiently large number of equations, N, the system (8) can be expressed in matrix form as:

$$\mathbf{x}'(Z) + A\mathbf{x}(Z) = \mathbf{0} \tag{9a}$$

subject to the initial condition

$$\mathbf{x}(0) = \mathbf{f} \tag{9b}$$

where

$$\mathbf{x}(Z) = \{\bar{\tilde{\theta}}_1(Z), \bar{\tilde{\theta}}_2(Z), \ldots, \bar{\tilde{\theta}}_N(Z)\}^T \tag{9c}$$

$$\mathbf{f} = \{\bar{f}_1, \bar{f}_2, \ldots, \bar{f}_N\}^T \tag{9d}$$

$$\mathbf{A} = \{\delta_{kj}\mu_k^2 + i\Omega a_{kj}^*\}, \quad k, j = 1, 2, \ldots, N \tag{9e}$$

and

$$\delta_{kj} = \begin{cases} 0, & k \neq j \\ 1, & k = j \end{cases} \tag{9f}$$

and prime denotes differentiation with respect to Z.

The matrix $\mathbf{A}$ is complex, full, and non-hermitian. To solve system (9), we first assume that $\mathbf{A}$ has a full set of N linearly independent eigenvectors and we seek solutions of the form:

$$\mathbf{x}(Z) = \mathbf{v}\, e^{-\lambda Z} \tag{10}$$

where the scalar λ and the constant vector $\mathbf{v}$ are yet to be determined. Introducing equation (10) into equation (9a), we obtain the following algebraic problem:

$$(\mathbf{A} - \lambda \mathbf{I})\mathbf{v} = \mathbf{0}, \tag{11}$$

where I is the $N \times N$ identity matrix. Equation (11) corresponds to the problem of finding eigenvalues and respective eigenvectors of the complex matrix $\mathbf{A}$.

Therefore, if $\mathbf{x}^{(1)}, \mathbf{x}^{(2)}, \ldots, \mathbf{x}^{(N)}$ form a fundamental set of solutions for equation (9a), the solution of the initial value problem (9) can be written as a linear combination of these fundamental solutions in the form

$$\mathbf{x}(Z) = c_1 \mathbf{x}^{(1)}(Z) + \cdots + c_N \mathbf{x}^{(N)}(Z) \tag{12a}$$

or,

$$\mathbf{x}(Z) = c_1 \mathbf{v}^{(1)} e^{-\lambda_1 Z} + \cdots + c_N \mathbf{v}^{(N)} e^{-\lambda_N Z}. \tag{12b}$$

In terms of the fundamental matrix of the system [8], $\Psi(Z)$, this result is written more compactly as:

$$\mathbf{x}(Z) = \mathbf{\Psi}(Z)\mathbf{c}. \tag{12c}$$

To determine the unknown integration constants $\mathbf{c}$, we constrain the solution given by equation (12c) to satisfy the initial condition, and obtain the following linear system of algebraic equations

$$\Psi(0)\mathbf{c} = \mathbf{f}. \tag{13a}$$

This result establishes the constants $c_1, c_2, \ldots, c_N$, where

$$\Psi(0) = \{v_k^{(j)}\}, \quad k, j = 1, 2, \ldots, N. \tag{13b}$$

Once equations (11) and (13) have been numerically solved using the standard subroutine packages, such as Ref. [9], the transformed temperature distribution, $\tilde{\theta}(R, Z)$, is determined. Noting that the dimensionless temperature $\theta(R, Z, \tau)$ is related to the function $\tilde{\theta}(R, Z)$ by equation (3), then the dimensionless wall heat flux is evaluated by utilizing the following expression:

$$-\left.\frac{\partial \tilde{\theta}(R, Z)}{\partial R}\right|_{R=1} = -\sum_{k=1}^{\infty} \frac{1}{N_k^{1/2}} \psi'(\mu_k, 1) \bar{\tilde{\theta}}_k(Z) \tag{14}$$

and the fluid bulk temperature by utilizing

$$\tilde{\theta}_{av}(Z) = (n+1) \int_0^1 R^n U(R) \tilde{\theta}(R, Z)\, dR \tag{15a}$$

or,

$$\tilde{\theta}_{av}(Z) = (n+1) \sum_{k=1}^{\infty} \bar{f}_k \bar{\tilde{\theta}}_k(Z), \tag{15b}$$

where

$$\bar{\bar{\theta}}_1(Z) \cong c_1 v_1^{(1)} e^{-\lambda_1 Z} + \cdots + c_N v_1^{(N)} e^{-\lambda_N Z}$$
$$\vdots \qquad (16)$$
$$\bar{\bar{\theta}}_N(Z) \cong c_1 v_N^{(1)} e^{-\lambda_1 Z} + \cdots + c_N v_N^{(N)} e^{-\lambda_N Z},$$

with $\bar{\bar{\theta}}_k(Z)$ assumed negligible for $k > N$.

Since equations (14) and (15b) define complex quantities, the final solutions can be conveniently expressed in polar coordinates as:

$$-\left.\frac{\partial\theta(R,Z,\tau)}{\partial R}\right|_{R=1} = A_h(Z)\exp\{i[\Omega\tau+\phi_h(Z)]\} \qquad (17)$$

$$\theta_{\mathrm{av}}(Z,\tau) = A_b(Z)\exp\{i[\Omega\tau+\phi_b(Z)]\}, \qquad (18)$$

where A's and ϕ's are, respectively, amplitudes and phase lags of oscillations with respect to the inlet condition, evaluated from the real and imaginary parts of equations (14) and (15b).

Lowest-order solution

The solution of the system of equations (8) presents some difficulty because they are coupled through the independent variables. However, if the coefficients matrix were a diagonal matrix, the system would be uncoupled. Furthermore, the solution of this linear system is mathematically equivalent to the problem of diagonalizing a matrix and decoupling the system. From inspection of the coefficient matrix, we observe that, especially for smaller values of Ω, the diagonal elements are dominant with respect to non-diagonal elements. This fact suggests a way of obtaining a straightforward approximate solution, that is, by letting $j = k$ in the summation of equation (8a). The approximate decoupled system then becomes:

$$\frac{d\bar{\bar{\theta}}_{l,k}(Z)}{dZ} + (\mu_k^2 + i\Omega a_{kk}^*)\bar{\bar{\theta}}_{l,k}(Z) = 0 \qquad (19a)$$

$$\bar{\bar{\theta}}_{l,k}(0) = \bar{f}_k, \quad k = 1,2,\ldots, \qquad (19b)$$

which has the explicit solution

$$\bar{\bar{\theta}}_{l,k}(Z) = \bar{f}_k e^{-\mu_k^2 Z} e^{-i\Omega a_{kk}^* Z}. \qquad (20)$$

Then, the complete lowest-order solution is given by:

$$\tilde{\theta}_l(R,Z) = \sum_{k=1}^{\infty} \frac{1}{N_k^{1/2}} \bar{f}_k \psi(\mu_k, R) e^{-\mu_k^2 Z} \cdot \exp(-i\Omega a_{kk}^* Z). \qquad (21)$$

Noting that the dimensionless temperature $\theta(R,Z,\tau)$ is related to the function $\tilde{\theta}(R,Z)$ by equation (3), then the wall heat flux and the fluid bulk temperature are determined by utilizing the following expressions:

$$-\left.\frac{\partial\tilde{\theta}_l(R,Z)}{\partial R}\right|_{R=1} = -\sum_{k=1}^{\infty} \frac{1}{N_k^{1/2}} \bar{f}_k \psi'(\mu_k, 1) e^{-\mu_k^2 Z} \cdot \exp(-i\Omega a_{kk}^* Z) \qquad (22)$$

$$\tilde{\theta}_{l,\mathrm{av}}(Z) = (n+1)\sum_{k=1}^{\infty} (\bar{f}_k)^2 e^{-\mu_k^2 Z} \exp(-i\Omega a_{kk}^* Z). \qquad (23)$$

Again, the complex quantities above can be conveniently expressed in polar form, with reference to the inlet condition, as in equations (17) and (18). The accuracy of this simple, approximate solution is considered in the following section.

RESULTS AND DISCUSSION

We consider the thermal response of laminar flow inside a parallel-plate channel and circular tube subjected to periodic variation of the inlet temperature. In order to obtain numerical results for the variations of the dimensionless wall heat flux and bulk fluid temperature, the algebraic eigenvalue problem (11) and the linear system of algebraic equations (13a) are solved by using appropriate subroutines from the IMSL package [9], and $\bar{\bar{\theta}}_k$'s are determined. We have taken $N \leqslant 40$, which proved to be more than sufficient for the desired convergence in the range of interest. The values of the dimensionless frequency of inlet oscillations considered included $\Omega = 0.0, 0.1, 0.5, 1.0, 2.0$ and 5.0, where $\Omega = 0.0$ corresponds to the classical Graetz problem. We also considered the lowest-order

Table 1. Comparison of first three eigenvalues λ_k and diagonal elements a_{kk} of the coefficients matrix, **A**, for different values of dimensionless frequency, Ω, in a parallel-plate channel

	k	1		2		3	
Ω		Real	Imag.	Real	Imag.	Real	Imag.
0.5	λ_k	(0.1885E+01,	0.3806E+00)	(0.2143E+02,	0.4768E+00)	(0.6232E+02,	0.5080E+00)
	a_{kk}	(0.1885E+01,	0.3806E+00)	(0.2143E+02,	0.4768E+00)	(0.6232E+02,	0.5080E+00)
1.0	λ_k	(0.1886E+01,	0.7613E+00)	(0.2143E+02,	0.9537E+00)	(0.6232E+02,	0.1016E+01)
	a_{kk}	(0.1885E+01,	0.7613E+00)	(0.2143E+02,	0.9537E+00)	(0.6232E+02,	0.1016E+01)
2.0	λ_k	(0.1889E+01,	0.1522E+01)	(0.2143E+02,	0.1907E+01)	(0.6232E+02,	0.2032E+01)
	a_{kk}	(0.1885E+01,	0.1523E+01)	(0.2143E+02,	0.1907E+01)	(0.6232E+02,	0.2032E+01)
5.0	λ_k	(0.1912E+01,	0.3804E+01)	(0.2145E+02,	0.4769E+01)	(0.6232E+02,	0.5080E+01)
	a_{kk}	(0.1885E+01,	0.3806E+01)	(0.2143E+02,	0.4768E+01)	(0.6232E+02,	0.5080E+01)

solution, equation (21), since its explicit form is useful for fast, approximate numerical evaluation for most practical purposes. To illustrate the validity of this concept, we present in Table 1, the first three eigenvalues and diagonal elements of the coefficients matrix A, for different values of Ω, for flow inside a parallel-plate channel. Since the problems of diagonalizing a matrix and solving a system of linear first-order differential equations are mathematically equivalent, the comparison of eigenvalues and diagonal elements demonstrates the importance of non-diagonal elements of matrix A in the process of decoupling the present system. From this table we notice that for $\Omega = 0.5$ and 1.0, the eigenvalues and diagonal elements are very close; only for $\Omega = 5.0$ the first eigenvalue is considerably perturbed by the increasing magnitude of non-diagonal elements. This comparison provides some confidence on the appropriateness of the lowest-order solution, especially for smaller values of Ω.

In Fig. 1, we present results for the amplitude and phase lag of bulk temperature with respect to the inlet condition for a parallel-plate channel, plotted as a function of the axial coordinate. The amplitudes for both the complete solution and lowest-order approximation are practically coincident over the range considered. Amplitudes are attenuated with the axial distance along the channel. Also shown in this figure are the phase lags for $\Omega = 0.1$, 0.5, 1.0 and 2.0, which increase significantly with Ω. If the increase in the parameter Ω is interpreted as a decreasing value of the thermal diffusivity of the fluid, α, the fluid heat capacity is regarded as a factor controlling the fluid temperature variation. That is, the thermal storage in the fluid "delays" the information sensed at each axial location downstream with respect to the inlet disturbance carried by the thermal wave. The predictions

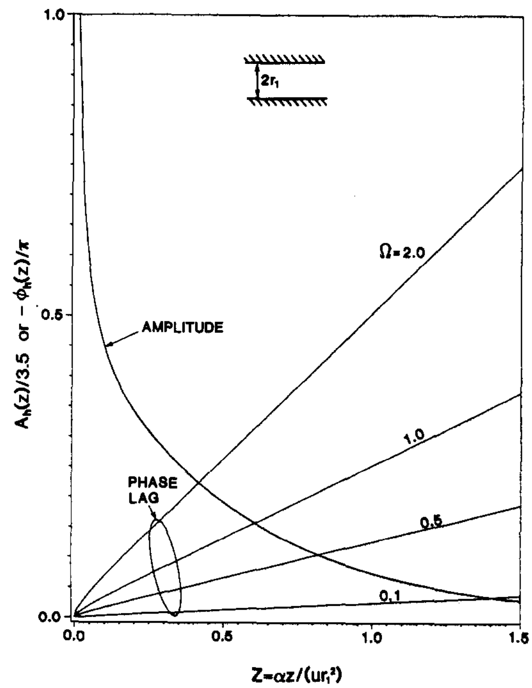

FIG. 2. Amplitude and phase lag of wall heat flux for flow inside a parallel-plate channel with different values of Ω.

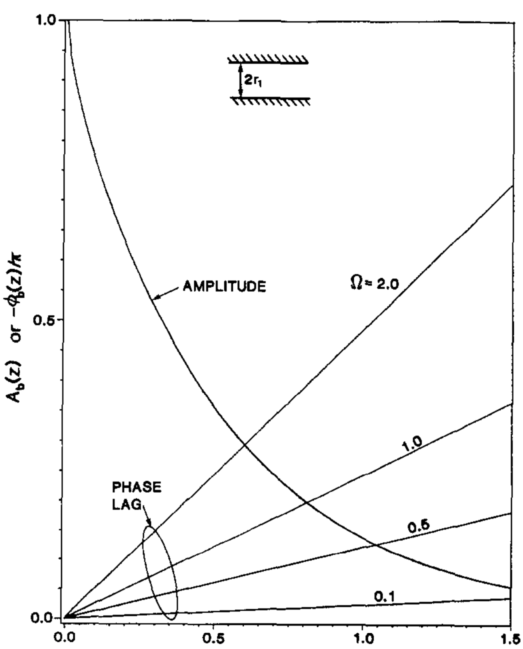

FIG. 1. Amplitude and phase lag of bulk temperature for flow inside a parallel-plate channel with different values of Ω.

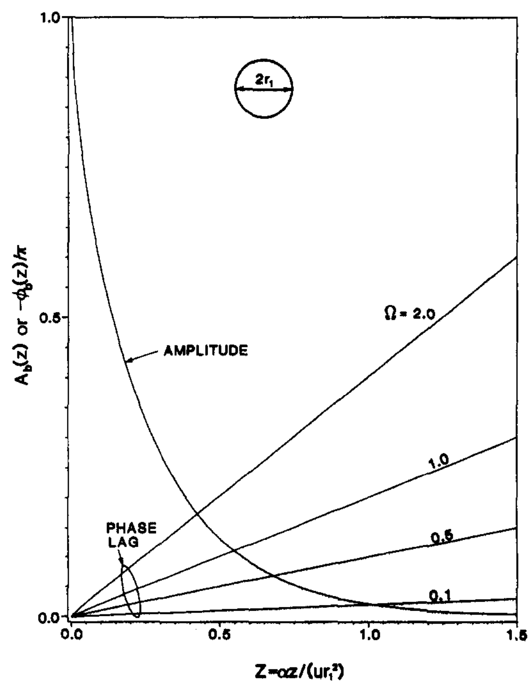

FIG. 3. Amplitude and phase lag of bulk temperature for flow inside a circular duct with different values of Ω.

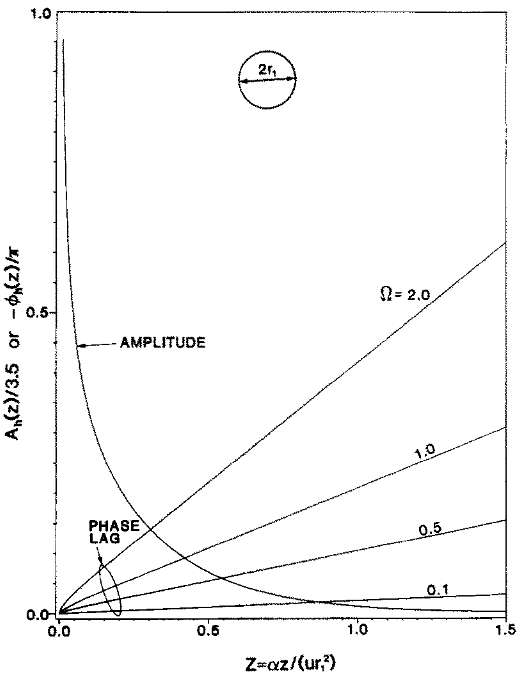

FIG. 4. Amplitude and phase lag of wall heat flux for flow inside a circular duct with different values of Ω.

through the lowest-order solution demonstrate the same trends, and the results are almost coincident as far as graphical presentation is concerned.

Figure 2 shows the amplitudes and phase lags obtained from the complete solution for the dimensionless wall heat flux for flow inside a parallel-plate channel. Amplitudes for both solutions are again practically coincident over the range of Ω considered here. Phase lags demonstrate the same behavior previously observed; the lowest-order solution, however, appears to be less accurate in this case, indicating that the nondiagonal elements become relatively more important in the derivatives of the temperature field. If an increasing value of Ω is interpreted as decreasing values of the fluid thermal conductivity, then less heat is transferred through the channel cross section to the boundaries at each axial location, which again "delays" the information sensed at the boundaries, carried by the thermal wave, and increases the phase lag with respect to the inlet disturbance at each position along the duct.

Figures 3 and 4, respectively, show the amplitudes and phase lags for dimensionless bulk temperature and wall heat flux for flow inside a circular tube. General trends similar to those observed for the case of parallel-plate duct, including those on the behavior of the approximate lowest-order solution, can be repeated here for the case of a circular tube. We note that amplitudes and phase lags, for bulk temperature and wall heat flux, are somewhat larger for the parallel-plates geometry than for circular tube.

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REFERENCES

1. S. Kakaç and Y. Yener, Transient laminar forced convection in ducts, in *Low Reynolds Number Flow Heat Exchangers* (edited by S. Kakaç, R. K. Shah and A. E. Bergles), pp. 205–227. Hemisphere, New York (1983).
2. M. D. Mikhailov, Mathematical modelling of heat transfer in single duct and double pipe exchangers, in *Low Reynolds Number Flow Heat Exhangers* (edited by S. Kakaç, R. K. Shah and A. E. Bergles), pp. 137–164. Hemisphere, New York (1983).
3. E. M. Sparrow and F. N. de Farias, Unsteady heat transfer in ducts with time-varying inlet temperature and participating walls, *Int. J. Heat Mass Transfer* **11**, 837–853 (1968).
4. S. Kakaç and Y. Yener, Frequency response analysis of transient turbulent forced convection for timewise variation of inlet temperature, in *Turbulent Forced Convection in Channels and Bundles* (edited by S. Kakaç and D. B. Spalding), Vol. 2, pp. 865–880. Hemisphere, New York (1979).
5. S. Kakaç and Y. Yener, Exact solution of the transient forced convection energy equation for timewise variation of inlet temperature, *Int. J. Heat Mass Transfer* **16**, 2205–2214 (1973).
6. M. N. Özişik and R. L. Murray, On the solution of linear diffusion problems with variable boundary condition parameters, *J. Heat Transfer* **C96**, 48 (1974).
7. M. D. Mikhailov and M. N. Özişik, *Unified Analysis and Solutions of Heat and Mass Diffusion*. Wiley, New York (1984).
8. W. E. Boyce and R. C. DiPrima, *Elementary Differential Equations*, 3rd edn. Wiley, New York (1977).
9. *IMSL Library*, Edition 7, GNB Building, 7500 Ballaire Blvd., Houston, Texas 77036 (1979).

CONVECTION LAMINAIRE FORCEE DANS DES CONDUITS AVEC VARIATION PERIODIQUE DE LA TEMPERATURE D'ENTREE

Résumé—La convection laminaire forcée avec variation périodique de la température d'entrée est étudiée à la fois pour les canaux entre plans parallèles et les tubes circulaires. La technique de transformation intégrale généralisée est employée pour réduire le problème original à un système d'équations différentielles linéaires du premier ordre qui est ensuite résolu en utilisant le problème conjoint de valeur propre à matrice complexe. Des amplitudes et des déphasages par rapport à la condition d'entrée sont déterminés pour la température moyenne du fluide et pour le flux thermique à la paroi; les résultats sont présentés sous forme graphique en fonction de la distance axiale adimensionnelle le long des conduits pour différentes valeurs de la fréquence réduite des oscillations à l'entrée.

ERZWUNGENE LAMINARE KONVEKTION IN STRÖMUNGSKANÄLEN BEI PERIODISCH SICH ÄNDERNDEN EINTRITTSSTEMPERATUREN

Zusammenfassung—Die erzwungene laminare Konvektion bei periodisch sich ändernden Eintrittstemperaturen wird sowohl in Kanälen aus parallelen Platten als auch in Rohren von Kreisquerschnitt untersucht. Mit Hilfe des allgemeinen Integral-Transformations-Verfahrens erhält man ein System linearer Differentialgleichungen 1. Ordnung, das unter Verwendung des Eigenwertproblems der zugehörigen komplexen Matrix gelöst wird. In Abhängigkeit der Randbedingungen am Eintritt werden Amplitude und Phasenverschiebung der mittleren Fluidtemperatur und der Wärmestromdichte ermittelt. Die Ergebnisse werden grafisch als Funktion der dimensionslosen Lauflänge im Kanal dargestellt, und zwar für unterschiedliche Werte der dimensionslosen Frequenz der Eintrittsoszillationen.

ЛАМИНАРНАЯ ВЫНУЖДЕННАЯ КОНВЕКЦИЯ ВНУТРИ КАНАЛОВ С ПЕРИОДИЧЕСКИМ ИЗМЕНЕНИЕМ ТЕМПЕРАТУРЫ НА ВХОДЕ

Аннотация—Изучается ламинарная вынужденная конвекция с периодическим изменением температуры на входе в плоско-параллельных каналах и круглых трубах. Для сведения начальной задачи к системе линейных дифференциальных уравнений первого порядка, которые затем решаются как задача на собственные значения, используется обобщенная методика интегрального преобразования. Амплитуды и отставания по фазе относительно условий на входе определялись для температуры ядра потока на стенке; результаты представлены в графической форме как функции безразмерного осевого расстояния вдоль каналов для различных значений безразмерной частоты колебаний на входе.